

# COMMUNITY UNIT SCHOOL DISTRICT 200

## Investigating Algebra Middle School Grade 8

- 1. Subject Expectation (State Goal 6)**      **Students will be able to demonstrate and apply an understanding of numbers and their operations, including meaning and relationships.**

<b>Essential Learning 1 (Learning Standard A) (Learning Standard D)</b>	<b>Understand numbers, ways of representing numbers, relationships among numbers, and number systems</b>
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Critical Content	6.A.3	a. find the prime factorization of a number and express it in exponential form
	6.A.3	b. apply understanding of fractions as division when simplifying complex fractions written in fraction, mixed number, or decimal form example: $(1 \frac{2}{3})/4$ , $2.3/5$
	6.B.3a	
	*	c. use absolute value in computations
	6.A.3	d. express large and small numbers using scientific notation including negative exponents
	6.A.3	e. represent percents smaller than 1% and larger than 100% in fraction or decimal form
	6.C.3a	f. convert repeating decimals to fractions using the algebraic method emphasizing properties of equality
	*	g. understand the structure of the real number system including recognizing subsets of the rationals and the distinguishing features of rational and irrational numbers
	6.A.3	h. find the approximate location of any rational number including negatives on a number line
	6.B.3b	i. recognize irrational numbers as non repeating and non-terminating decimals
	6.A.3	j. differentiate between an exact and an approximate representation of a number
	6.A.3	k. recognize that any positive number has two square roots, a positive and a negative

<b>Essential Learning 2 (Learning Standard B)</b>	<b>Understand meanings of operations and how they relate to one another</b>
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Critical Content	6.B.3c	a. apply number properties including commutative, associative, distributive, equality, transitive, identities, inverses, zero property and closure to rational numbers
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- 6.B.3a b. evaluate numerical expressions using order of operations, emphasizing grouping symbols such as absolute value, fraction bar, nested grouping symbols
- \* c. find GCF and LCM of numbers expressed in exponential form Example : GCF of  $2^4 \cdot 5^2$  and  $2^3 \cdot 5^4$  is  $2^3 \cdot 5^2$
- 6.B.3b d. apply factors, multiples, GCF, or LCM in real life situations using 2 or more events
- 6.B.3c e. understand negative exponents as a reciprocal relationship
- 6.C.3b f. recognize proportions as a special class of equations which may be solved using cross products
- 6.D.3 g. use proportional reasoning to
  - solve problems using similar figures, discount, sales tax, percent, percent increase/decrease, scale drawings and other proportions
  - calculate unit rates
  - solve equations such as  $\frac{2}{3} - \frac{x}{4} = 0$
- 6.D.3 h. know equivalent percent, decimal and fraction forms (families of eighths, sixths as well as halves, thirds, fourths, fifths, tenths) and work flexibly within to solve problems
- 6.D.3 i. solve problems using the percent equation ( $P=BR$ )
  - discount, sales tax
  - percent increase/decrease
  - connect to algebra using distributive property in sale price and mark up
    - payment =  $(1.00 + .06)\text{price}$
    - sale price =  $(1.00 - .20)\text{price}$
- 6.B.3c j. calculate square roots of perfect squares and approximate square roots of other numbers to the nearest whole number
- 6.A.3 k. locate an irrational number (represented as decimal or as a square root) on a number line
- 6.A.3 l. differentiate between the exact name and an approximate name of an irrational number
- \* m. represent an irrational number in geometric sense
  - ex- diagonal of a unit square
  - sides of squares of a given area using dot paper or geoboard
- \* n. solve equations of form  $x^2=24$
- 6.C.3a o. apply square roots to solve geometric problems

<b>Essential Learning 3 (Learning Standard C)</b>	<b>Compute fluently and make reasonable estimates</b>
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| Critical Content | <ul style="list-style-type: none"> <li>6.C.3a a. develop and use mental math strategies appropriate to 8<sup>th</sup> grade concepts, especially those dealing with the distributive property           <ul style="list-style-type: none"> <li>• 43,464 divided by 2000</li> <li>• <math>8 * 53 = 8( 50 + 3 )</math></li> </ul> </li> <li>6.B.3a b. develop competency in all integer operations both mentally and using paper and pencil</li> <li>6.A.3 c. evaluate numbers in exponential notation emphasizing negative exponents</li> </ul> |
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- 6.A.3 d. convert between scientific and decimal notation for all numbers emphasizing negative exponents
- 6.B.3c e. convert between percent, decimal, and fractions with percents less than 1% and greater than 100%
- 6.B.3c f. recognize perfect squares from 169 through 400

<b>Essential Learning 4</b>	<b>Choose appropriate technology/tools</b>
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| Critical Content | * | <ul style="list-style-type: none"> <li>a. select appropriate methods and tools, according to the context, for computing           <ul style="list-style-type: none"> <li>• mental computation</li> <li>• estimation</li> <li>• calculators (scientific or graphic)</li> <li>• manipulatives and models such as dot paper, geoboards, counting chips, number lines and graph paper</li> <li>• paper and pencil</li> </ul> </li> </ul> |
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<b>Essential Learning 5*</b>	<b>Recognize the connections between number sense and other math strands</b>
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| Critical Content | 8.D.3a  | a. use a variable to represent a missing quantity in a percent proportion or equation                                    |
|                  | 10.A.3b | b. use experimental probability activities to apply fraction, decimal, and percent skills                                |
|                  | 10.C.3b | c. use dot paper or geoboards to represent the diagonal of a unit square (irrational number)                             |
|                  | *       | d. use dot paper or geoboards to create squares with irrational side lengths by tilting (connect to Pythagorean theorem) |

<b>Essential Learning 6*</b>	<b>Construct and communicate convincing arguments</b>
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| Critical Content | * | <ul style="list-style-type: none"> <li>a. demonstrate correct usage of the vocabulary related to number sense: irrational, rational, real number, non-terminating, non-repeating</li> </ul>  |
|                  | * | <ul style="list-style-type: none"> <li>b. make and test conjectures and form generalizations about number sense           <ul style="list-style-type: none"> <li>• explain the algorithms for all operations with rational numbers</li> <li>• verbally and mathematically defend whether a number is rational or irrational</li> </ul> </li> <li>c. show evidence that computational results using rational numbers and percents are correct and/or that estimates are reasonable</li> </ul> |

**2. Subject Expectation  
(State Goal 7)**

**The student will be able to estimate, make and use measurements of objects, quantities and relationships and determine acceptable levels of accuracy.**

<b>Essential Learning 1 (Learning Standard A)</b>	<b>Understand measurable attributes of objects and the units, systems, and process of measurement</b>
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| Critical Content | * | a. estimate and calculate area and perimeter of irregular or composite shapes <ul style="list-style-type: none"><li>• distance round the “track”</li><li>• area of a “house” shape (triangle and rectangle)</li><li>• space between 2 concentric circles</li></ul> |
|                  | * | b. investigate relationships between measurement and similarity <ul style="list-style-type: none"><li>• corresponding angles and sides of similar figures</li><li>• perimeters of corresponding shapes are proportional to corresponding side lengths</li></ul>    |

<b>Essential Learning 2 (Learning Standard B) (Learning Standard C)</b>	<b>Apply appropriate techniques, tools and formulas to determine measurements</b>
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| Critical Content | 7.C.3b | a. use formulas for circumference and area of circles |
|                  | 7.C.3b | b. calculate surface area of prisms and cylinders     |
|                  | 7.C.3b | c. calculate volume of prisms and pyramids            |
|                  | 7.C.3b | d. calculate volume of cones and cylinders            |

<b>Essential Learning 3</b>	<b>Recognize the connections between measurement, other math strands, and other curricular areas</b>
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| Critical Content | *      | a. explore and apply relationship between area and perimeter <ul style="list-style-type: none"><li>• find the maximum area of a dog pen given 32 yards of fencing</li></ul>  |
|                  | 9.C.3a | b. understand the concept of the constant $\pi$ as the ratio of the circumference to the diameter of the circle <ul style="list-style-type: none"><li>• use software such as Geometer’s sketchpad software</li></ul> |

<b>Essential Learning 4</b>	<b>Construct and communicate convincing arguments and proofs to solve problems</b>
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| Critical Content | * | a. demonstrate appropriate usage of vocabulary related to measurement                            |
|                  | * | b. develop logical arguments to justify conclusions about topics such as volume and surface area |

**3. Subject Expectation  
(State Goal 8)**

**The student will be able to use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems and predict results.**

<b>Essential Learning 1 (Learning Standard A) (Learning Standard B)</b>	<b>Understand patterns, relations, and functions</b>
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| Critical Content | 8.A.3b | a. translate written expressions that involve multiple operations into algebraic expressions   |
|                  | 8.A.3b | b. translate algebraic expressions into verbal expressions   |
|                  | 8.B.3  | c. represent and analyze patterns and functions using words, tables, graphs, and symbolic rules  |
|                  | 8.B.3  | d. relate and compare different forms of representation for a relationship   |
|                  | 8.B.3  | e. differentiate between a relation and a function and identify the domain and range   |
|                  | 8.B.3  | f. identify functions as linear or non-linear and contrast their properties from tables, graphs, or equations  |
|                  | 8.B.3  | g. use graphing technology and algebraic methods to analyze and predict linear relationships and make generalizations from linear patterns                             |
|                  | 8.B.3  | h. explore writing a simple algebraic equation for a line of best-fit from a scatterplot   |
|                  | *      | i. describe, extend, create and make generalizations about geometric and numeric patterns including arithmetic sequences, geometric sequences, and Fibonacci sequences |

<b>Essential Learning 2 (Learning Standard C)</b>	<b>Represent and analyze mathematical situations and structures using algebraic symbols</b>
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| Critical Content | 8.A.3a | a. evaluate algebraic expressions using order of operations including nested groupings and exponential expressions   |
|                  | 8.C.3  |  |
|                  | 8.D.3c | <ul style="list-style-type: none"> <li>• <math>3x^2</math>, <math>-3x^2</math>, <math>(-3x)^2</math></li> </ul>  |
|                  | 8.A.3a | b. apply the distributive and transitive, symmetric, and reflexive properties to algebraic situations  |
|                  | 8.A.3b | c. develop proficiency in recognizing equivalent expressions and equations   |
|                  | 8.A.3b | d. use number properties to simplify an algebraic expression or equation and justify steps   |
|                  | 8.D.3a | e. express mathematical relationships using equations and inequalities   |
|                  | 8.A.3a | f. demonstrate how to solve and check multi-step equations having variables on one or two sides using rational numbers <ul style="list-style-type: none"> <li>• <math>2x - 5 = -10</math></li> <li>• <math>(\frac{2}{3})x + 5 = -4</math></li> <li>• <math>-3x + 2 = 3(3x - 1)</math></li> </ul> |
|                  | 8.D.3a | g. solve literal equations <ul style="list-style-type: none"> <li>• given <math>Ax + By = C</math> solve for y</li> </ul>  |

- \* h. demonstrate how to solve and check multi-step equations that have no real number solutions or are identities
- 8.A.3a i. demonstrate how to solve and graph inequalities with rational coefficients
- 8.A.3b
- 8.D.3a j. solve linear systems graphically, incorporating real life situations

<b>Essential Learning 3 (Learning Standard D)</b>	<b>Use mathematical models to represent and understand quantitative relationships and solve problems</b>
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| Critical Content | 8.A.3b | a. | translate sentences and word problems into algebraic equations and inequalities and solve   |
|                  | *      | b. | identify the independent and dependent variables in a contextual problem  |
|                  | 8.B.3  | c. | use t-tables to graph functions with an emphasis on linear functions  |
|                  | *      | d. | investigate and develop the concept of constant rate, or slope, and the concept of y-intercept and explore their interactions   |
|                  | 8.B.3  | e. | graph and write equations for horizontal and vertical lines   |
|                  | 8.B.3  | f. | graph linear functions <ul style="list-style-type: none"> <li>• by making a table</li> <li>• by using slope and y-intercept</li> <li>• by using x- and y- intercepts</li> <li>• by using a point and a slope</li> </ul>             |
|                  | *      | g. | find an equation in standard or slope-intercept form of a line using <ul style="list-style-type: none"> <li>• point and slope</li> <li>• two points</li> <li>• table</li> <li>• graph</li> <li>• best-fit line from data</li> </ul> |
|                  | *      | h. | write an equation of a line that passes through a given point and is parallel or perpendicular to the graph of a given equation   |
|                  | 8.D.3a | i. | graph linear inequalities in one or two variables   |

<b>Essential Learning 4*</b>	<b>Use problem solving to analyze change in real life situations</b>
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| Critical Content | *     | a. | describe the story context on the basis of a graph that displays changes over time; using resources such as <ul style="list-style-type: none"> <li>• motion detector (CBR) programs</li> <li>• Visual Approach to Algebra by Van Dyke</li> </ul>  |
|                  | *     | b. | investigate how a change in one variable relates to a change in a second related variable using tables, graphs, equations, and graphing calculators <ul style="list-style-type: none"> <li>• identify correlation</li> <li>• explore non-linear functions such as <math>y = x^2</math>, <math>y = 1/x</math></li> </ul> |
|                  | 8.C.3 | c. | explore direct variation ( $y = kx$ ) as it relates to the real world   |

<b>Essential Learning 5</b>	<b>Choose appropriate technology/tools for algebraic representation</b>
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| Critical Content | 8.B.4a | a. use manipulatives, such as Algebra Gear, AlgeTiles, etc to solve linear equations                                |
|                  | 8.B.3  | b. use technology, including graphing calculators, to look at the relationships among tables, graphs, and equations |

<b>Essential Learning 6*</b>	<b>Recognize the connections between algebra and other math strands</b>
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| Critical Content | 9.C.3b | a. use equations to set up geometrical relationships; ( e.g., an angle’s supplement is = twice its complement)  |
|                  | 6.D.3  | b. use ratio and proportion to solve problems   |
|                  | 8.A.3a | c. apply the order of operations  |
|                  | 8.A.3a | d. apply the commutative and associative properties for addition and multiplication and the distributive property of multiplication over addition/subtraction |
|                  | 8.B.3  | e. find, describe, extend and create patterns such as those found in Pascal’s triangle, the Fibonnaci sequence, and express these symbolically                |
|                  | *      | f. use manipulatives to make geometric representations of the distributive property   |

<b>Essential Learning 7*</b>	<b>Construct and communicate convincing arguments and proofs to solve problems</b>
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| Critical Content | *      | a. use appropriate algebraic vocabulary including number properties (identities, inverses, associative, commutative, distributive), equality properties ( reflexive, symmetric, transitive), coefficient, quantity, constant, function, domain, range, slope, intercept, correlation, coefficient, constant, dependent and independent variable, simplifying, systems of equations |
|                  | *      | b. demonstrate correct usage of the language of patterns such as linear and nonlinear, multiples   |
|                  | *      | c. use number and equality properties to justify steps in solving equations  |
|                  | *      | d. develop logical arguments to justify conclusions about topics such as transitives and varying rates of change   |
|                  | 8.D.3a | e. identify, symbolically represent, and test the rule used to generate a pattern  |

**4. Subject Expectation  
(State Goal 9)**

**The student will use geometric methods to analyze, categorize, and draw conclusions about points, lines, planes and space.**

<b>Essential Learning 1 (Learning Standard A) (Learning Standard B) (Learning Standard D)</b>	<b>Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships</b>
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Critical Content	9.B.3	a. describe, classify, and compare triangles and quadrilaterals (parallelograms and trapezoids) according to their side and angle properties
	9.B.3	b. determine the complement and/or supplement of a given angle (angle = $x$ ; complement = $90 - x$ ; supplement = $180 - x$ )
	9.D.3	c. develop and use the Pythagorean Theorem and its converse

<b>Essential Learning 2*</b>	<b>Specify locations and describe spatial relationships using coordinate geometry and (or) other representational systems</b>
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Critical Content	*	a. use coordinate geometry to describe reflections and translations
	9.A.3a	c. draw and compare 3-D geometric figures including cylinders and cones
	9.B.3	

<b>Essential Learning 3 (Learning Standard C)</b>	<b>Use visualization, spatial reasoning, and geometric modeling to solve problems</b>
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Critical Content	9.C.3b	a. explore and apply the relationships between parallel lines, transversals and angles
	9.C.3b	b. explore and apply relationships of vertical angles and linear pairs
	9.C.3a	c. explore relationships between number of sides and diagonals of polygons
	9.C.3b	d. explore and apply relationships among radius, circumference and area of circle <ul style="list-style-type: none"> <li>• (e.g., How does doubling the radius affect circumference and area?)</li> </ul>

<b>Essential Learning 4 (Learning Standard)</b>	
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Critical Content	9.C.3a	a. describe the effects of reflections, translations and dilations on the shape and size of the figure
	9.A.3b	b. dilate a polygon given a scale factor (expansion and contraction)

<b>Essential Learning 5*</b>	<b>Recognize the connections between geometry and other math strands</b>
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| Critical Content | 7.A.3b<br>9.D.3 | a. predict and apply formulas for volume of cone and pyramid<br>b. relate Pythagorean formula to algebraic situations<br>c. recognize that similarity is a proportional relationship |
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<b>Essential Learning 6 (Learning Standard C)</b>	<b>Construct convincing arguments and proofs to solve problems</b>
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| Critical Content | *<br>9.C.3a | a. use appropriate vocabulary in a given situation<br>b. construct, develop, and communicate arguments (informal proofs) about geometric figures and patterns |
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<b>Essential Learning 7</b>	<b>Choose appropriate technology/tools for geometric representations</b>
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| Critical Content | 7.A.3a<br>7.B.3<br>9.C.3b | a. use protractors, rulers, graph paper<br>b. use software such as Geometer's Sketchpad to investigate <ul style="list-style-type: none"> <li>• angles created by a transversal and parallel lines, vertical angles, and linear pair</li> <li>• transformations</li> </ul> |
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**5. Subject Expectation (State Goal 10)      The student will select, organize and analyze data using statistical methods; predict results; and interpret uncertainty-using concepts of probability.**

<b>Essential Learning 1 (Learning Standard A) (Learning Standard B)</b>	<b>Develop concepts of data collection and analysis</b>
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| Critical Content | 10.B.3  | a. represent 2-variable data relationships using scatterplots <ul style="list-style-type: none"> <li>• construct scatterplots with linear and non-linear relationships</li> <li>• draw approximate line of best-fit (e.g., use spaghetti on plots)</li> <li>• use graphing calculators to explore line of best-fit</li> <li>• determine if there is a positive, negative or no correlation</li> </ul> |
|                  | 10.A.3a | b. select, create and use appropriate graphical representations of data, including histograms and box-and-whisker plots   |
|                  | 10.A.3c | c. interpret quantitative measures from a histogram; e.g., <ul style="list-style-type: none"> <li>• How many studied at least 4 hours (to be determined via histogram)?</li> </ul>  |
|                  | 10.A.3b | d. understand the meaning of, and be able to identify or compute the minimum, the lower quartile, the median, the upper quartile, the interquartile range, the maximum and possible outliers of a data set.   |
|                  | 10.A.3a | e. construct box-and-whisker plots  |

- examine data spread
- use box-and-whisker plots to compare similar sets of data

<b>Essential Learning 2 (Learning Standard C)</b>	<b>Develop the concept of probability</b>
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| Critical Content | 10.C.3a | a. compute odds of simple events  |
|                  | 10.C.3a | b. compute probabilities of compound events using organized lists, tree diagrams, Fundamental Counting Principle  |
|                  | 10.C.3a | c. determine replacement (independent) and non-replacement (dependent) probabilities through data collection experiments, such as picking from a bag of marbles |
|                  | 10.C.3a | d. determine theoretical probabilities of independent, dependent, and mutually exclusive events using “and”, “or”, or “not”                                     |

<b>Essential Learning 3*</b>	<b>Choose appropriate tools for data collection and representation</b>
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| Critical Content | * | a. use manipulatives (such as spinners, number cubes, and counting chips) appropriate to activities   |
|                  | * | b. use graph paper to construct and analyze scatterplots  |
|                  | * | c. use technology, such as graphing calculators, to analyze data and generate box-and-whisker plots, scatterplots, and histograms and generate random numbers |

<b>Essential Learning 4*</b>	<b>Recognize the connections between data collection and probability and other math strands</b>
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| Critical Content | * | a. analyze and extrapolate conclusions from graphical representations |
|                  | * | b. relate line of best-fit to linear equations                        |

<b>Essential Learning 5*</b>	<b>Construct and communicate convincing arguments and proofs to solve problems</b>
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| Critical Content | a. demonstrate correct usage of the vocabulary related to data collection and probability  |
|                  | b. explain which measure of central tendency is most appropriate in a given context <ul style="list-style-type: none"> <li>• skewed salaries, home prices</li> </ul> |
|                  | c. analyze problem situations and make predictions about results, including fairness   |
|                  | d. develop logical arguments to justify the reason for a prediction  |