

# COMMUNITY UNIT SCHOOL DISTRICT 200

## Precalculus High School Intermediate Level

**Subject Expectation  
(State Goal 6)**      **The student will demonstrate and apply a knowledge and sense of numbers, including numeration and operations (addition, subtraction, multiplication, division), patterns, ratios and proportions.**

<b>Essential Learning 1 (Learning Standard A)</b>	<b>Demonstrate knowledge and use of numbers and their representations in a broad range of theoretical and practical settings</b>
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Critical Content	6.A.4	Identify and apply the associative, commutative, distributive and identity properties of real numbers, including special numbers such as pi and square roots
	6.A.5	Perform addition, subtraction and multiplication of complex numbers and graph the results in the complex plane <ul style="list-style-type: none"><li>• given a graph of a polynomial function, determine:<ul style="list-style-type: none"><li>- relative extrema</li><li>- absolute extrema</li><li>- intervals where the function is increasing, decreasing, or constant</li><li>- identify possible equations of the polynomial function</li></ul></li><li>• determine relative extrema of polynomial functions</li><li>• evaluate simple logarithms</li><li>• illustrate the relationship between second and third roots and powers of a number</li><li>• represent numbers in equivalent forms (e.g., exponential/logarithmic, radical/rational exponents)</li><li>• represent numerical intervals using correct notation</li></ul>

<b>Essential Learning 2 (Learning Standard B)</b>	<b>Investigate, represent and solve problems using number facts, operations (addition, subtraction, multiplication, and division) and their properties, algorithms and relationships</b>
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Critical Content	6.B.5	Identify, represent and apply numbers expressed in exponential, logarithmic and scientific notation using contemporary technology
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- recognize and use number system
  - natural
  - whole
  - integers
  - rational
  - irrational
- recognize and use field properties
  - closure
  - commutative
  - associative
- solve quadratic equations with real and imaginary solutions
- solve logarithmic equations
- graph exponential and logarithmic functions
- evaluate exponential and logarithmic expressions
- compare and contrast the properties of numbers and number systems, including the rational and the real and complex numbers
- determine an appropriate numerical representation of a problem situation, including roots and powers, if applicable
- judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities
- develop fluency in operations with real numbers using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases
- judge the reasonableness of numerical computations and their results
- compare and contrast the properties of numbers and number systems, including the complex numbers as solutions to quadratic equations that do not have real solutions
- simplify expressions using the field properties, order properties, and properties of equality for the set of real numbers
- use the field properties and properties of equality for the set of complex numbers
- identify, represent, and solve problems with numbers expressed in exponential, logarithmic, and scientific notations using technology
- solve problems using exponents and logarithms

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<b>Essential Learning 3 (Learning Standard C)</b>	<b>Compute and estimate using mental mathematics, paper-and-pencil methods, calculators and computers</b>
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Critical Content      6.C.5      Determine the level of accuracy needed for computations involving measurement and irrational numbers

- find a specific term of a sequence
- develop fluency in operations with real numbers using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases
- determine and explain whether exact values or approximations are needed in a variety of situations
- determine an appropriate number of digits to represent an outcome
- describe the role of rounding error in calculations
- determine the level of accuracy needed for computations involving measurement and irrational numbers
- use the correct number of digits in computation to achieve an appropriate unit when solving problems
- estimate an appropriate answer for a given term of a sequence

<b>Essential Learning 4</b>	<b>Learn and use appropriate problem solving strategies</b>
<b>(Learning Standard D)</b>	<b>Solve problems using comparison and quantities, ratios, proportions and percents</b>

Critical Content	6.D.5	<p>Solve problems involving loans, mortgages and other practical applications involving geometric patterns of growth</p> <ul style="list-style-type: none"> <li>• solve exponential and logarithmic equations: <ul style="list-style-type: none"> <li>- applications of exponential growth and decay</li> <li>- interest problems</li> </ul> </li> <li>• explain how ratios and proportions can be used to solve problems of percent, growth and error tolerance</li> <li>• explain the connection of percents to growth patterns, and probability</li> </ul>
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**Subject Expectation (State Goal 7)**      **The student will estimate, make and use measurements of objects, quantities, and relationships and determine acceptable levels of accuracy**

<b>Essential Learning 1</b>	<b>Measure and compare quantities using appropriate units, instruments and methods.</b>
<b>(Learning Standard A)</b>	

Critical Content	7.A.5	<p>Apply nonlinear scales (e.g., Richter, decibel, ph) to solve practical problems</p> <ul style="list-style-type: none"> <li>• draw angles of rotation in standard position given the measure of the angle or a description of a rotation</li> <li>• determine the degree or radian measure given a description of an angle of rotation</li> </ul>
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- find the measure of coterminal angles
- determine each angle measure in decimal degrees
- determine each angle measure in degree-minutes-seconds
- convert between degree and radian measure
- solve problems regarding arc length and areas of sectors without a graphing calculator
- with a graphing calculator:
  - convert between degree and radian measure
  - solve problems regarding arc length and areas of sectors
  - solve trigonometric equations using right angles
- choose units and scales that are appropriate for problem situations involving measurement
- \*
  - determine how the conversion of units of the dimensions of a figure affects the number of units of the other measures such as area and volume
  - convert angle measures between degrees and radians

<b>Essential Learning 2 (Learning Standard B)</b>	<b>Estimate measurements and determine acceptable levels of accuracy</b>
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| Critical Content | 7.B.5 | <ul style="list-style-type: none"> <li>• estimate the magnitude and directions of physical quantities (e.g., velocity, force, and slope)</li> <li>• determine answers to an appropriate degree of accuracy using significant digits</li> <li>• determine a reasonable estimate of measure for more complex problem situations</li> <li>• solve problems to a desired interval of accuracy</li> <li>• apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations</li> </ul> |
|                  | *     | <ul style="list-style-type: none"> <li>• analyze precision, accuracy, and approximate error in measurement situations</li> </ul>   |

<b>Essential Learning 3 (Learning Standard C)</b>	<b>Select and use appropriate technology, instruments and formulas to solve problems, interpret results and communicate findings</b>
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| Critical Content | 7.C.5a | Use dimensional analysis to determine units and check answers in applied measurement problems <ul style="list-style-type: none"> <li>• solve problems involving linear and angular velocity without a graphing calculator</li> <li>• solve problems involving linear and angular velocity</li> </ul> |
|                  | 7.C.5b | Determine how changes in one measure may affect other measures <ul style="list-style-type: none"> <li>• solve problems using indirect measurement by choosing appropriate technology, instruments, and/or formulas</li> </ul>  |

- check measurement computations using unit analysis
- explain the meaning of a measurement answer in context
- solve practical problems using non-linear scales
- set up and solve measurement conversions using multiple rates and conversion factors

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**Subject Expectation  
(State Goal 8)**

**The student will use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems and predict results.**

<b>Essential Learning 1 (Learning Standard A)</b>	<b>Describe numerical relationships using variables and patterns</b>
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| Critical Content | 8.A.5 | <p>Solve mathematical problems involving recursive patterns and use models that employ such relationships</p> <ul style="list-style-type: none"> <li>• without a graphing calculator: <ul style="list-style-type: none"> <li>- write a mathematical model given a set of data; <ul style="list-style-type: none"> <li>➤ linear</li> <li>➤ quadratic</li> </ul> </li> <li>- write equivalent forms of equations, inequalities, and systems of equations</li> <li>- simplify and evaluate expressions with rational exponents</li> <li>- simplify logarithmic expressions using: <ul style="list-style-type: none"> <li>➤ change of base</li> <li>➤ three properties of logarithms</li> </ul> </li> <li>- find a rule for a sequence</li> <li>- identify explicit or recursive formulas</li> <li>- determine whether a sequence is arithmetic or geometric</li> <li>- find geometric and arithmetic means</li> <li>- denote a series in sigma and expanded notation</li> <li>- find partial sums of series</li> <li>- find infinite sums of converging series</li> <li>- find infinite sums of converging series</li> <li>- use binomial theorem to expand a binomial</li> <li>- use binomial theorem to find a specific term of a binomial expansion</li> <li>- find limits of functions from: <ul style="list-style-type: none"> <li>➤ graph</li> <li>➤ equation</li> </ul> </li> <li>- find the derivative of a function using: <ul style="list-style-type: none"> <li>➤ definition</li> <li>➤ theorems</li> </ul> </li> </ul> </li> </ul> |
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- with a graphing calculator:
  - use a binomial theorem to expand a binomial
  - use binomial theorem to find a specific term of a binomial expansion
- represent and explain mathematical relationships using symbolic algebra
- model and describe slope as a constant rate of change
- explain the difference between constant and non-constant rate of change
- create an equation of a line of best fit used in Algebra I and Algebra II from a set of ordered pairs or set of data points
- simplify algebraic expressions using a variety of methods, including factoring
- \* • justify the results of symbol manipulations, including those carried out by technology
- \* • identify essential quantitative relationships in a situation and determine the class or classes of functions (e.g., linear, quadratic) that might model the relationships
- represent relationships arising from various contexts using algebraic expression
- \* • generalize patterns using explicitly defined and recursively-defined sequences
- translate between explicit and recursive forms of sequences where possible
- \* • symbolize growth patterns using variables
- explain the differences and similarities of different forms of growth formulas
- describe the limiting process using numerical analysis, graphs, and algebra
- \* • simplify algebraic expressions using exponential, logarithmic, and rational number techniques, including more advanced factoring

<b>Essential Learning 2 (Learning Standard B)</b>	<b>Interpret and describe numerical relationships using tables, graphs, and symbols</b>
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| Critical Content | 8.B.5 | <p>Use functions including exponential, polynomial, rational, parametric, logarithmic, and trigonometric to describe numerical relationships</p> <ul style="list-style-type: none"> <li>• use special angles to evaluate trigonometric expressions and solve equations</li> <li>• use special angles to evaluate trigonometric functions and solve equations</li> <li>• solve trigonometric equations using:           <ul style="list-style-type: none"> <li>- right angles</li> <li>- law of sines</li> <li>- law of cosines</li> </ul> </li> </ul> |
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- without a graphing calculator:
  - determine whether a relation is a function
  - determine the domain and range of a relation or function
  - graph functions on the Cartesian coordinate system
    - linear
    - quadratic
    - absolute value
    - greatest integer
    - piecewise
  - find inverse of a relation or function
  - determine if the inverse of a function is a function
  - form new functions by adding, subtracting, multiplying and dividing
  - form and evaluate composite functions
  - determine the symmetry of a graph (y-axis, x-axis, origin,  $y = x$ )
  - use symmetry to sketch a graph
  - determine whether a function is even or odd given a:
    - graph
    - equation
  - graph functions using reflections in the x-axis, y-axis, and the line  $y=x$
  - graph functions using translations and dilations
  - find the vertex and axis of symmetry of a quadratic function
  - sketch a graph of a quadratic function using the vertex and/or zeros
  - sketch a graph of a quadratic function using translations and dilations
  - determine the equation of each function from a graph using translations and dilations
  - graph an inverse, given a graph of a function
  - graph basic polynomial functions
  - determine whether a function is even or odd from:
    - graph
    - equation
  - given a rational function, find discontinuities using:
    - asymptotes
    - vertical
    - horizontal
    - slant
    - holes

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- graph rational functions (in factored form)
- graph simple exponential functions
- know the equations and graph the four conic sections from their equations
  - circle
  - ellipse
  - hyperbola
  - parabola
- find the center/vertex of each conic section from its equation
- set up parametric equations using real-world situations
- with a graphing calculator:
  - graph functions on the Cartesian coordinate system
    - linear
    - quadratic
    - absolute value
    - greater integer
    - piecewise
  - determine best-fit equation and predict outcomes from a set of data
    - linear
    - quadratic
  - determine the domain and range of a function
  - find the vertex and axis of symmetry of a quadratic function
  - graph parametric equations
  - solve application problems involving parametric equations
  - describe the relationships of the independent and dependent variables from a graph
  - graph polynomial functions
  - identify intervals where polynomial function is increasing, decreasing, or constant
  - interpret the role of the coefficients and constants on the graph of linear and quadratic functions given a set of equations
  - determine the effect of translations on linear relations
  - create and connect representations that are tabular, graphical, numeric, and algebraic from a set of data
  - recognize and describe the general shape and properties of the graphs of linear, absolute value, and quadratic functions
  - approximate and interpret rates of change from graphical and numerical data

- identify slope in an equation and from a table of values
- graph absolute values of linear functions on the Cartesian plane
- \* - fit an equation to data using a calculator
- interpret the overall relationship of two variables and connect it to one of the function families (linear, exponential, logarithmic or power) from a graph
- relate the effect of transformations on graphs and equations
- analyze functions by investigating domain, range, rates of change, intercepts, zeros, and asymptotes
- \* - describe and perform transformations such as arithmetically combining, composing, and inverting commonly used functions using technology to perform such operations on more complicated symbolic expressions
- relate functions to their inverses and their reflections over the line  $y = x$
- analyze functions and their graphs for symmetries
- use a variety of symbolic representations for functions and relations, including piecewise functions
- \* - describe the properties and features of any non-degenerate conic section from an equation or graph
- write an equation for conic sections from a graph

<b>Essential Learning 3 (Learning Standard C)</b>	<b>Solve problems using systems of numbers and their properties</b>
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| Critical Content | 8.C.5 | <p>Use polynomial, exponential, logarithmic, rational and periodic functions to model situations</p> <ul style="list-style-type: none"> <li>• with a graphing calculator:             <ul style="list-style-type: none"> <li>- solve application problems involving angular and linear velocity</li> </ul> </li> <li>• without a graphing calculator:             <ul style="list-style-type: none"> <li>- divide polynomials using synthetic division</li> <li>- apply the remainder and factor theorems</li> <li>- set up exponential growth and decay equations</li> <li>- determine domains of logarithmic functions</li> <li>- describe and compare the properties of linear and quadratic functions</li> </ul> </li> </ul> |
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- interpolate and extrapolate to solve problems using systems of numbers
- solve problems using translations and dilations on basic functions
- solve application problems with and without a graphing calculator
- \* - describe and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions
- identify and explain the relationship between arithmetic/geometric sequences and linear/exponential functions
- \* - describe the relationship of a model of a problem to the real problem
- apply sequences and their properties to solve real problems
- model and solve real problems using mathematical functions and relations

<b>Essential Learning 4 (Learning Standard D)</b>	<b>Use algebraic concepts and procedures to represent and solve problems</b>
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| Critical Content | 8.D.5 | <p>Formulate and solve nonlinear equations and systems including problems involving inverse variation and exponential and logarithmic growth and decay</p> <ul style="list-style-type: none"> <li>• with a graphing calculator: <ul style="list-style-type: none"> <li>- find roots of a quadratic function</li> <li>- solve quadratic equations</li> <li>- solve and graph quadratic inequalities</li> <li>- solve and graph polynomial inequalities</li> <li>- solve and graph rational inequalities</li> </ul> </li> <li>• without a graphing calculator: <ul style="list-style-type: none"> <li>- find roots of a quadratic function by <ul style="list-style-type: none"> <li>➤ factoring</li> <li>➤ completing the square</li> <li>➤ quadratic formula</li> </ul> </li> <li>- solve polynomial equations</li> <li>- solve and graph quadratic inequalities</li> <li>- solve and graph polynomial inequalities</li> <li>- solve and graph rational inequalities</li> <li>- solve equivalent forms of equations, inequalities, and systems of equations with fluency – mentally or with paper and pencil in simple cases and using technology in all cases</li> <li>- interpret and use functions as a geometric representation of linear and non-linear relationships</li> <li>- solve simple quadratic equations using algebraic or graphical representations</li> </ul> </li> </ul> |
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  - solve problems using equations of exponential, logarithmic and logistic growth
  - solve problems using rational equations and inequalities
  - set up and solve problems of non-linear growth
  - solve problems using linear programming

**Subject Expectation  
(State Goal 9)**

**The student will use geometric methods to analyze, categorize, and draw conclusions about points, lines, planes and space**

**Essential Learning 1  
(Learning Standard A)**      **Demonstrate and apply geometric concepts involving points, lines, planes and space**

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| Critical Content | 9.A.5 | Use geometric figures and their properties to solve problems in arts, the physical and life sciences and the building trades, with and without the use of technology <ul style="list-style-type: none"> <li>• use and apply the distance formula without a graphing calculator</li> <li>• use and apply the midpoint formula without a graphing calculator</li> <li>• calculate distance, midpoint coordinates, and slope using coordinate geometry</li> <li>• calculate distance, midpoint coordinates, and slope using coordinate geometry</li> </ul> |
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**Essential Learning 3  
(Learning Standard C)**      **Construct convincing arguments and proofs to solve problems**

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| Critical Content | 9.C.5b | Apply physical models, graphs, coordinate systems, networks and vectors to develop solutions in applied contexts (e.g., Bus routing, areas of irregular shapes, describing forces and other physical quantities) <ul style="list-style-type: none"> <li>• find a counter-example to disprove a conjecture</li> <li>• extend the ideas of formal and informal proof to non-geometric situations</li> </ul> |
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**Essential Learning 4  
(Learning Standard D)**      **Use trigonometric ratios and circular functions to solve problems**

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| Critical Content | 9.D.5 | Analyze and solve problems involving periodic patterns (e.g., sound waves, tide variations) using circular functions and communicate results orally and in writing |
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- without a graphing calculator:
  - determine the trigonometric functions of angles using the unit circle
  - determine the reference angle for a given angle of rotation
  - use special angles to evaluate trigonometric expressions
  - use a reference angle to determine the trigonometric functions for the given angle
  - know, understand and use the trigonometric identities
    - reciprocal
    - Pythagorean
    - ratio
    - odd-even
  - use sum, difference, half and double-angle identities to determine the exact value of given trigonometric expressions
  - prove other identities using other identities
  - solve trigonometric equations and inequalities using known identities
  - determine the period and amplitude of a periodic function given a graph
  - find period, amplitude, and phase shifts of the 6 basic trigonometric functions given :
    - graph
    - equation
  - graph equations of the 6 basic trigonometric functions
  - define the inverses of the 6 basic trigonometric functions
  - evaluate expressions using inverse trigonometric functions
  - solve equations using inverse trigonometric functions
- with a graphing calculator:
  - evaluate trigonometric expressions
  - solve trigonometric equations and inequalities using known identities
  - determine whether an equation is an identity
  - graph the 6 basic trigonometric functions
  - solve equations using inverse trigonometric functions
  - evaluate expressions using inverse trigonometric functions
  - solve application problems
  - understand the domain and range of inverse trigonometric functions

- determine distances and angle measures using indirect measurement and properties of right triangles
- solve problems using 45-45-90 and 30-60-90 triangles
- relate circular functions, arcs, and radian measure to triangle trigonometry and degree measure
- simplify expressions and solve problems using trigonometric identities
- solve equations using circular functions
- identify key characteristics of graphs of trigonometric functions and their inverses
- graph trigonometric functions using translations and dilations
- graph a given trigonometric function using its characteristics (e.g., period, and amplitude)

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**Subject Expectation  
(State Goal 10)**

**The student will collect, organize and analyze data using statistical methods; predict results; and interpret uncertainty-using concepts of probability.**

<b>Essential Learning 1 (Learning Standard A)</b>	<b>Organize, describe and make predictions from existing data</b>
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Critical Content

- analyze two-variable data for linear or quadratic fit
- make decisions based on data, including the relationships of correlation and causation
- interpolate, extrapolate, and make predictions from given information

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<b>Essential Learning 3 (Learning Standard C)</b>	<b>Determine, describe and apply the probabilities of events</b>
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Critical Content

- 10.C.5a Compute conditional probabilities and the probabilities of independent events
- with a graphing calculator:
    - solve problems involving mutually exclusive and independent events
    - solve problems involving conditional probability
- 10.C.5b Compute probabilities in counting situations involving permutations and combinations.

- without a graphing calculator:
  - review the basic counting principles
  - identify dependent and independent events
- with a graphing calculator:
  - set up and solve permutation problems
  - set up and solve combination problems
  - determine sample spaces
  - calculate probabilities that events will occur